UDC 532.526

THREE-DIMENSIONAL BOUNDARY LAYER IN AN IONIZED GAS IN CHEMICAL EQUILIBRIUM

PMM Vol. 41, № 6, 1977, pp. 1024-1032

S. N. KAZEIKIN, G. A. TIRSKII, and Iu. D. SHEVELEV

(Moscow)

(Received January 17, 1977)

A system of equations is given for the three-dimensional laminar boundary layer in a multicomponent gas in the state of equilibrium, in the presence of ionization reactions and under the condition of quasineutrality. The external electromagnetic fields and radiative energy transfer are both assumed absent. A method of integration is proposed, based on the method of consecutive approximations. The transport coefficients and the associated functions characterizing the variability of the physical and chemical properties of the gas are approximated across the boundary layer. In the first locally self-similar approximation, simple formulas are derived for the surface friction and heat exchange coefficients. Examples of computation in a flow of equilibrated partly ionized air past a cone with a spherically blunted nose at an angle of attack, and comparison is made with the frozen case.

1. Let us consider a flow in a three-dimensional boundary layer of an N-component, quasineutral equilibrium gas, in which N_r independent reactions take place including the ionization reactions. Following [1], we shall write the reactions taking place in the gas, in the form

$$A_i = \sum_{j=N_r+1}^{N} \nu_{ij} A_j - Q_i^*, \quad i = 1, 2, \ldots, N_r \tag{1.1}$$

where ν_{ij} are the stoichiometric coefficients, A_i is the chemical symbol of the i-th component and Q_i^* is the molar heat of the i-th reaction. We assign a constant number N to the electronic component.

The system of equations of the multicomponent partly ionized three-dimensional laminar boundary layer in chemical equilibrium can be written, in the absence of external electromagnetic fields and radiative energy transfer, in the form

$$\frac{\partial}{\partial \xi}\left(\rho \sqrt{\frac{g}{g_{11}}}\, u\right) + \frac{\partial}{\partial \eta}\left(\rho \sqrt{\frac{g}{g_{22}}}\, w\right) + \sqrt{g}\, \frac{\partial \rho v}{\partial \zeta} = 0 \tag{1.2}$$

$$\rho L c_k^* + \frac{\partial I_k^*}{\partial \zeta} = 0, \quad k = N_r + 1, \ldots, N$$

$$Lu + A_1 u^2 + A_2 w^2 + A_3 uw = \frac{A_4}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \zeta}\left(\mu \frac{\partial u}{\partial \zeta}\right)$$

$$Lw + B_1 u^2 + B_2 w^2 + B_3 uw = \frac{B_4}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \zeta}\left(\mu \frac{\partial w}{\partial \zeta}\right), \quad \frac{\partial p}{\partial \zeta} = 0$$

$$\rho L H = \frac{\partial}{\partial \zeta}\left\{\frac{\mu}{\sigma'}\left[\frac{\partial H}{\partial \zeta} + (\sigma' - 1)\frac{\partial}{\partial \zeta}\frac{U^2}{2} + \right.\right.$$

$$\sum_{k=N_r+1}^{N-2}\left(\frac{\sigma'}{\mu}b_k I_k^* + a_k\frac{\partial c_k^*}{\partial\zeta}\right)\Big]\Big\}$$

$$H = \sum_{j=N_r+1}^{N} c_{jp}^* h_j + \frac{U^2}{2} - \sum_{k=1}^{N_r} c_k Q_k, \quad h_j = \int_0^T c_{pj}dT + h_j^0,$$

$$Q_k = \frac{Q_k^*}{m_k}$$

$$c_j^* = c_j + \sum_{k=1}^{N_r}\nu_{kj}\frac{m_j}{m_k}c_k, \quad I_j^* = I_j + \sum_{k=1}^{N_r}\nu_{kj}\frac{m_j}{m_k}I_k,$$

$$j = N_r+1, \ldots, N$$

$$L = \frac{u}{\sqrt{g_{11}}}\frac{\partial}{\partial\xi} + \frac{w}{\sqrt{g_{22}}}\frac{\partial}{\partial\eta} + v\frac{\partial}{\partial\zeta}$$

The first equation of (1.2) represents the equation of continuity, the second describes the diffusion of the elements, the third, fourth and fifth are equations of motion of the gas and the sixth is the energy equation. Thermal diffusion and the influence of the diffusing heat capacities [1] are both neglected. The system (1.2) is closed by the Stefan - Maxwell relations, the conditions of equilibrium and the equation of state of the gas mixture

$$\rho\frac{\partial x_k}{\partial\zeta} = m\sum_{i=1}^{N}\frac{I_i}{m_i}\left[\frac{x_k}{D_{ki}} - \delta_{ki}\sum_{l=1}^{N}\frac{x_l}{D_{il}} + \frac{x_k e_k}{L_1}\sum_{j=1}^{N}\frac{x_j}{D_{ij}}(e_i - e_j)\right] \qquad (1.3)$$

$$L_1 = \sum_{i=1}^{N} x_i e_i^2, \quad k = 1, 2, \ldots, N$$

$$\frac{1}{x_i}\prod_{j=N_r+1}^{N} x_j^{\nu_{ij}} = \frac{K_i(T)}{p^{\nu_i}}, \quad \nu_i = \sum_{j=N_r+1}^{N}\nu_{ij} - 1, \quad i = 1, \ldots, N_r \qquad (1.4)$$

$$p = \rho\frac{RT}{m}, \quad m = \sum_{k=1}^{N} x_k m_k \qquad (1.5)$$

Here ξ, η and ζ denote an orthogonal coordinate system in which the ζ -axis is directed along the normal to the body surface in such a way that the surface $\zeta = 0$ coincides with the body surface, and the ξ - and η -axes are directed along the surface of the body; g_{11} and g_{22} denote the components of the metric tensor and $g = g_{11}g_{22}$; u, v and w are the ξ -, ζ - and η - components of the mean mass velocity vector $\bar{U}$; p, ρ and T denote the pressure, density and absolute temperature of the mixture; m is the molecular weight of the mixture; c_i, x_i, m_i and e_i are the mass and molar concentration, molecular weight and electric charge of the i -th component; c_j^* is the mass concentration of the j -th element; I_i is the projection of the mass diffusion flux of the i -th component on the ζ -axis; I_j^* is the projection of the mass diffusion flux of the j -th element on the ζ -axis; D_{ij} are the binary diffusion coefficients; h_i, h_i^0 and c_{pi} denote the specific enthalpy, specific heat of formation and heat capacity of the i -th component;

H is the total enthalpy of the mixture; the index e denotes the parameters of the outer boundary of the boundary layer; μ is the viscosity of the mixture;and $\sigma' = \mu c_p' / \lambda'$ is the effective Prandtl number where c_p' and λ' are the effective heat capacity and effective heat conductivity coefficient of the equilibrium mixture. The formulas for calculating c_p' and λ' and the coefficients

$$a_j,\ b_j\ (j = N_r + 1,\ \ldots,\ N - 2)$$

which appear in the energy equation, were all given in [1]. The summation in the energy equation is carried out up to $N - 2$, since the N-th term vanishes by virtue of (1.7) and the $(N - 1)$-th term can be eliminated using the identity (1.6) (see below).

Figure 1 depicts the variation in the effective Prandtl number for air in equilibrium. The curves numbered 1 to 5 in Figs. 1-3 correspond to the pressures of 0.01, 0.1, 1, 10 and 100 atmospheres. Nine components

$$O_2,\ N_2,\ NO,\ O^+,\ N^+,\ NO^+,\ O,\ N,\ E$$

were considered and O, N and E were assumed to be independent. Such a model of air will have a single coefficient a_j and one b_j, Fig. 2 depicts these coefficients for oxygen.

The coefficients A_i and B_i $(i = 1, \ldots, 4)$ are determined by the geometry of the body and the external flow [2].

The Stefan-Maxwell relations (1.3) take into account the electric field appearing as the result of separation of the charged components. The magnitude of the field is found from the condition of quasineutrality of the gas.

Equations (1.4) represent the Guldberg—Waage conditions for the chemical reactions, and the Saha equations for the ionization reactions.

The system (1.2)-(1.5) must be supplemented by the indentities

$$\sum_{k=1}^{N} x_k = \sum_{k=1}^{N} c_k = 1, \quad \sum_{k=1}^{N} I_k = 0 \qquad (1.6)$$

$$\sum_{j=N_r+1}^{N} c_j^* = 1, \quad \sum_{j=N_r+1}^{N} I_j^* = 0$$

The conditions of quasineutrality of the gas and of the absence of an electric current across the boundary layer can be reduced, for a single ionization, under the condition that the electrically neutral components and electrons are regarded as the independent elements, to the form

$$c_N^* = 0,\ I_N^* = 0 \qquad (1.7)$$

The systems (1.2)-(1.5) with (1.6) taken into account, represents a closed system of $6 + 2N + 2(N - N_r)$ independent equations with unknowns p, ρ, u, w, v, H, $c_1, \ldots, c_N$, $I_1, \ldots, I_N$, $c_{N_r+1}^*, \ldots, c_N^*$, $I_{N_r+1}^*, \ldots, I_N^*$.

The boundary conditions at the outer boundary of the boundary layer and at the impermeable wall, are

$$u \to u_e(\xi, \eta),\ w \to w_e(\xi, \eta),\ H \to H_e = \text{const} \qquad (1.8)$$

$$c_j^* \to c_{je}^* = c_{j\infty}^* = \text{const} \quad \text{when } \zeta \to \infty$$

$$u = w = v = 0, \; T(\xi, \eta, 0) = T_w(\xi, \eta), \; I_j^* = 0 \quad \text{when } \zeta = 0$$

Having solved the problem with boundary conditions (1.8), we can find the viscous frictional stress distribution at the surface of the body, and the total convective heat flux towards the wall

$$\tau_{11} = \mu \frac{\partial u}{\partial \zeta}\bigg|_{\zeta=0}, \quad \tau_{22} = \mu \frac{\partial w}{\partial \zeta}\bigg|_{\zeta=0} \tag{1.9}$$

$$I_q = -\lambda' \frac{\partial T}{\partial \zeta}\bigg|_{\zeta=0} = -\frac{\mu}{\sigma'}\left(\frac{\partial H}{\partial \zeta} + \sum_{j=N_r+1}^{N-2} a_j \frac{\partial c_j^*}{\partial \zeta}\right)_{\zeta=0} \tag{1.10}$$

2. Let us consider the flow of a four-component gas past a body. The gas contains molecules, ions, atoms and electrons; dissociation and ionization reaction: both take place in the gas. We denote the gas components by M, I, A and E respectively. We choose the atoms and electrons as the elements. Then the reactions taking place in the gas can be written in the form

$$M \rightleftarrows 2A, \quad I \rightleftarrows A - E$$

In the present case we have $N = 4$, $N_r = 2$. Such a model enables us e.g. to study the flow of air past a body at temperatures ranging from normal to $15\,000° - 16\,000°$. Indeed, nitrogen and oxygen which are the main components of air have sufficiently similar properties. Moreover the thermodynamic functions of nitrogen and oxygen averaged with regard to the composition can be used as thermodynamic functions of the components.

From the conditions (1.7) of quasineutrality and absence of electric current and from (1.6), follows

$$c_A^* \equiv 1, \quad I_A^* \equiv 0 \tag{2.1}$$

The form of the system (1.2)-(1.5) differs in the present case from that of the system of equations of a one-component viscous compressible heat conducting three-dimensional boundary layer [3] only in the fact that the coefficients λ, c_p and μ for the one-component gas are replaced by the coefficients λ', c_p' and μ for the equilibrium reaction mixture. Naturally, the system must be supplemented by the relations from which these coefficients can be computed.

Let us now pass from the coordinate ζ to the self-similar coordinate λ and introduce the dimensionless normalized variables

$$\lambda = \sqrt{\frac{u_e}{\mu_e \rho_e a}} \int_0^\zeta \rho d\zeta \tag{2.2}$$

$$u = u_e(\xi, \eta) E(\xi, \eta, \lambda), \quad w = \beta(\xi, \eta) u_e(\xi, \eta)(G + \varphi E)$$

$$\rho v = \sqrt{\frac{\mu_e \rho_e u_e}{a}}\left[K - \frac{a}{\sqrt{g_{11}}} E \frac{\partial \lambda}{\partial \xi} - \frac{a\beta}{\sqrt{g_{22}}}(G + \varphi E)\frac{\partial \lambda}{\partial \eta}\right]$$

$$\varphi = \frac{w_e}{\beta u_e}, \quad \theta = \frac{H - H_w}{H_e - H_w}$$

where $\alpha(\xi, \eta)$ and $\beta(\xi, \eta)$ are functions, arbitrary for the time being.

Let us integrate the system of equations obtained twice with respect to λ, from λ to ∞ and from 0 to λ, taking into account the boundary conditions. This yields a system of integro-differential equations the solution of which, together with the boundary conditions for the normalized functions, is equivalent to the solution of the initial system of equations with boundary conditions (1.8). We shall solve this system using the method of consecutive approximations just as in the case of frozen gas [4]. Assume that the n-th approximation is known. Substituting this into the equations of the system we obtain the $(n+1)$-th approximation. We ensure that the $(n+1)$-th approximation satisfies the boundary conditions by introducing the controlling functions.

$$E^{(n+1)} = E(\xi, \eta, \lambda/\sqrt{\delta^{(n+1)}}), \quad G^{(n+1)} = b^{(n+1)}G(\xi, \eta, \lambda/\sqrt{\delta^{(n+1)}})$$
$$\theta^{(n+1)} = E^{(n+1)} - d^{(n+1)}F(\xi, \eta, \lambda/\sqrt{\delta^{(n+1)}}) \qquad (2.3)$$

The equations of the system yield an equation for the unknown controlling functions δ, b and d by making $\zeta \to \infty$.

Using the iteration process, we can find the quantities proportional to the frictional components and enthalpy gradient at the wall.

3. Let us consider the locally self-similar case [3,4]. The system of equations for the $(n+1)$-th approximation has the form

$$E^{(n+1)} = -\delta^{(n)}(A_1^{(n)} + b^{(n)}B_1^{(n)} + b^{(n)^2}C_1^{(n)}) \qquad (3.1)$$
$$G^{(n+1)} = -\delta^{(n)}(A_2^{(n)} + b^{(n)}B_2^{(n)} + b^{(n)^2}C_2^{(n)})$$
$$\theta^{(n+1)} = \delta^{(n)}(A_3^{(n)} + b^{(n)}B_3^{(n)} + d^{(n)}C_3^{(n)} + b^{(n)}d^{(n)}D_3^{(n)}) + E_3^{(n)}$$

The controlling functions are found from the system of algebraic equations

$$\delta^{(n)}(A_{1\infty}^{(n)} + b^{(n)}B_{1\infty}^{(n)} + b^{(n)^2}C_{1\infty}^{(n)}) = 1$$
$$A_{2\infty}^{(n)} + b^{(n)}B_{2\infty}^{(n)} + b^{(n)^2}C_{2\infty}^{(n)} = 0 \qquad (3.2)$$
$$\delta^{(n)}(A_{3\infty}^{(n)} + b^{(n)}B_{3\infty}^{(n)} + d^{(n)}C_{3\infty}^{(n)} + b^{(n)}d^{(n)}D_{3\infty}^{(n)}) + E_{3\infty}^{(n)} = 1$$

The coefficients $A_i^{(n)}, B_i^{(n)}, \ldots$ represent dual integrals the determination of which requires the knowledge of the behavior of the parameters ρ_e/ρ, $1/l$, σ'/l across the boundary layer $(l = \mu\rho/(\mu_e\rho_e))$. Typical examples of the profiles ρ_e/ρ and $1/l$ are given in [5], and their behavior is described well by the formulas

$$\rho_e/\rho = 1 + (\rho_e/\rho_w - 1)Z_{-1}^{1.2}(\zeta), \quad 1/l = 1 + (1/l_w - 1)Z_{-1}^{1.4}(\zeta)$$

Here and henceforth we use the functions $Z_m(\zeta)$ of the type

$$Z_{-1}(\zeta) = \exp(-\zeta^2), \quad Z_m(\zeta) = \frac{A_m}{m!}\int_0^{\zeta}(\zeta - t)^m \exp(-t^2)\,dt$$
$$m = 0, 1, \ldots$$

where A_m are found from the condition $Z_m(0) = 1$).

Computing the heat flux towards the body at the stagnation point and comparing it with the results of [5, 6], gives the following relation connecting σ' / l with the coordinate ζ:

$$\sigma' / l = \sigma_e' + (\sigma_w' / l_w - \sigma_e') Z_{-1}^{1.3}(\zeta)$$

We obtain the solution of the problem in the first approximation using the following expressions as the zero approximation:

$$E^{(0)} = 1 - Z_0(\zeta),\ G^{(0)} = b^{(0)}(\xi, \eta)[Z_0(\zeta) - Z_{-1}(\zeta)]$$
$$\theta^{(0)} = 1 - Z_0(\zeta) + d^{(0)}[Z_0(\zeta) - Z_{-1}(\zeta)]$$

The coefficients of the system (3.2) which yields the controlling functions are given in the zero approximation by

$$A_{1\infty}^{(0)} = -0.25P_1^* + 0.0075N_1^* + (1/l_w - 1)(0.0061N_1^* - 0.149P_1^*) - 0.417N_1^* \rho_e/\rho_w - 0.318N_1^*(1/l_w - 1)\rho_e/\rho_w$$
$$B_{1\infty}^{(0)} = 0.104P_2^* - 0.194N_2^* + (1/l_w - 1)(0.0674P_2^* - 0.126N_2^*)$$
$$C_{1\infty}^{(0)} = N_2^*[0.048 + 0.0364(1/l_w - 1)]$$
$$A_{2\infty}^{(0)} = M_1^*[0.0075 + 0.0061(1/l_w - 1) - 0.417\rho_e/\rho_w - 0.318(1/l_w - 1)\rho_e/\rho_w]$$
$$B_{2\infty}^{(0)} = -0.311P_1^* - 0.194M_3^* - (1/l_w - 1)(0.1535P_1^* - 0.126M_3^*)$$
$$C_{2\infty}^{(0)} = 0.1P_2^* + 0.048M_2^* + (1/l_w - 1)(0.0527P_2^* + 0.0364M_2^*)$$
$$A_{3\infty}^{(0)} = P_1^*(0.0973\sigma_e' + 0.153\sigma_w'/l_w),\ B_{3\infty}^{(0)} = -P_2^*(0.0347\sigma_e' + 0.0688\sigma_w'/l_w),\ D_{3\infty}^{(0)} = -P_2^*(0.046\sigma_e' + 0.054\sigma_w'/l_w)$$
$$C_{3\infty}^{(0)} = P_1^*(0.152\sigma_e' + 0.158\sigma_w'/l_w),\quad E_{3\infty}^{(0)} = \frac{1 - \sigma_w'}{(1 - t_0)k}(1 + \beta^2\varphi^2)$$
$$(l_w = \mu_w\rho_w / (\mu_e\rho_e),\ t_0 = H_w / H_e,\ k = 2H_e / u_e^2)$$

where the coefficients $N_1^*, N_2^*, M_1^*, M_2^*, P_1^*, P_2^*$ depend on the parameters of the outer flow and on the geometry of the body [2]. Having found $\delta^{(0)}, b^{(0)}$ and $d^{(0)}$ from (3.2), we can obtain the dimensionless components of the friction coefficient at the wall and the enthalpy gradient

$$-l_w \frac{\partial E}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\left\{N_1^*\left[0.808\left(1 - \frac{\rho_e}{\rho_w}\right) - 0.798\right] - 0.234P_1^* + b^{(0)}(0.113P_2^* - 0.209N_3^*) + 0.0709N_2^* b^{(0)2}\right\} \quad (3.3)$$

$$-l_w \frac{\partial G}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\left\{M_1^*\left[0.808\left(1 - \frac{\rho_e}{\rho_w}\right) - 0.798\right] - 0.209b^{(0)}(P_1^* + M_3^*) + 0.0709b^{(0)2}(P_2^* + M_2^*)\right\}$$

$$-\frac{l_w}{\sigma_w}\frac{\partial \theta}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\{-P_1^*(0.234 + 0.209d^{(0)}) + b^{(0)}P_2^*(0.113 + 0.0709d^{(0)})\}$$

4. Consider the flow of equilibrium air past a cone with a spherically blunted nose at an angle of attack. We choose the coordinate system at the surface in such a manner, that the coordinate ξ is directed along the generatrix of the cone and is measured from the leading point, and the coordinate η denotes the angle between the meridian plane passing through the given point and the windward spreading line.

We compute the outer flow using the data given in [7] for the pressure and velocity at the cone surface in a homogeneous perfect gas. The temperature, the composition of the mixture and the density were found from the solution of the system of equations determining T and c_i $(i = 1, 2, \ldots, N)$

$$H - \sum_{j=N_r+1}^{N} c_{je}^* h_j + \sum_{k=1}^{N_r} c_k Q_k - \frac{U^2}{2} = 0 \qquad (4.1)$$

$$\frac{1}{x_i} \prod_{j=N_r+1}^{N} x_j^{\nu_{ij}} = \frac{K_i(T)}{p^{\nu_i}}, \qquad i = 1, \ldots, N_r$$

$$\alpha_j + \sum_{i=1}^{N_r} \alpha_{ji} x_i - x_j = 0, \qquad j = N_r + 1, \ldots, N$$

$$\alpha_j = \frac{c_j^*}{m_j}\Big(\sum_{i=N_r+1}^{N} \frac{c_i^*}{m_i}\Big)^{-1}, \qquad \alpha_{ji} = \alpha_j \nu_i - \nu_{ij}, \qquad j = N_r + 1, \ldots, N$$

Here H_e and c_{je}^* are obtained from the conditions in an unperturbed gas, since from (1.8) we have $H_e = H_\infty = \text{const}$ and $c_{je}^* = c_{j\infty}^*$. When H, U, p and c_j^* $(j = N_r + 1, \ldots, N)$ are specified, the system (4.1) contains $N + 1$ independent equations with $N + 1$ unknown $c_1, \ldots, c_N$, T. This system was solved using the Newton's method. Having found the composition and temperature, we can obtain the molecular weight of the mixture and hence the density from the equation of state. The transport coefficients at the boundaries were computed as in [4].

We obtain the following expressions for the longitudinal and transverse components of the local coefficient of friction and the Nusselt number both defined in [4]:

$$C_{f1}\sqrt{\text{Re}} = \sqrt{\delta^{(0)}}\,(0.234P_1^* - 0.01N_1^*), \qquad C_{f2}\sqrt{\text{Re}} = \frac{w_e}{u_e} C_{f1}\sqrt{\text{Re}} \qquad (4.2)$$

$$\frac{1}{\sigma_e}\frac{\text{Nu}}{\sqrt{\text{Re}}} = \sqrt{\delta^{(0)}}\, P_1^* (0.234 + 0.209 d^{(0)})$$

The formulas (4.2) were obtained for high speeds of flight $(M_\infty \geqslant 20)$, taking into account the smallness of the secondary flow and of the ratio ρ_e / ρ_w; this is true for a sufficiently cold wall.

The coefficients P_1^* and N_1^* appear and are investigated in [2] in the course of study of a frozen boundary layer.

Since the external nonviscous flow and the coordinate system were taken as identical, the coefficient N_1^* has the same value in the equilibrium case as in the frozen case. The coefficient P_1^* which depends on the variation of the parameter $\mu_e\rho_e$, becomes different on moving away from the stagnation point.

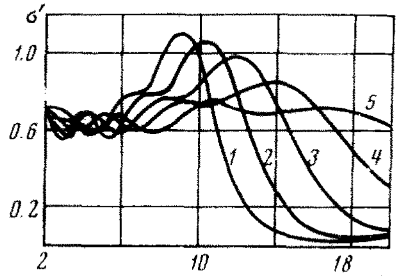

Fig. 1

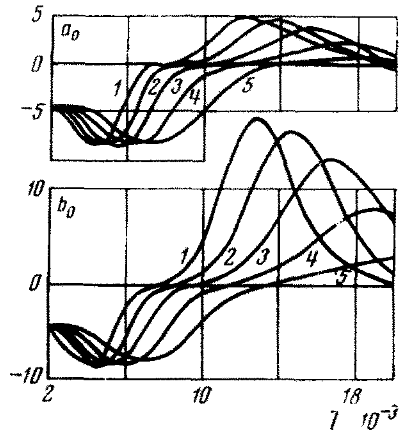

Fig. 2

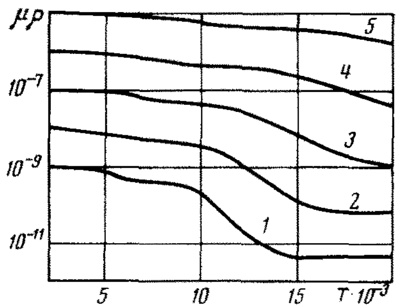

Fig. 3

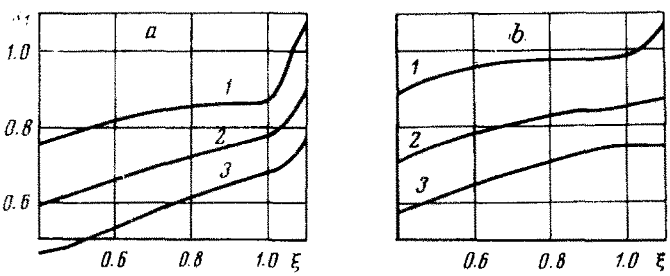

Fig. 4

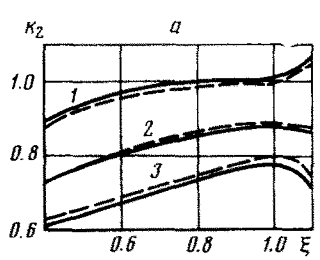

Fig. 5

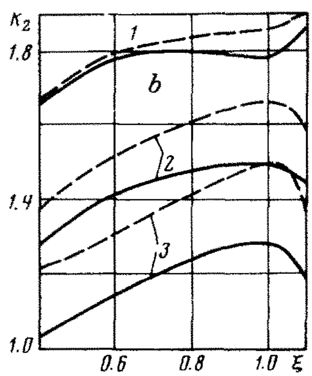

Figure 3 depicts the dependence of the parameter $\mu\rho$ on temperature and pressure for the case of air in equilibrium. The influence of equilibrium on the values of the controlling functions is shown by

$$\delta^{(0)} = [0.101\ P_1^* - 0.0014\ N_1^* + (0.149\ P_1^* - 0.0061\ N_1^*)\ /\ l_w]^{-1} \tag{4.3}$$

$$d^{(0)} = \frac{1-(1-\sigma')(1+\beta^2\varphi^2)/[k(1-t_0)] - P_1^*\delta^{(0)}(0.0973\sigma_e' + 0.1536\sigma_w'/l_w)}{P_1^*\delta^{(0)}(0.152\sigma_e' + 0.158\sigma_w'/l_w)}$$

The transverse and longitudinal components of the local coefficient of friction for the equilibrium boundary layer differ in value by several percent from the components of the coefficient of friction in the frozen gas. Figure 4 depicts the variation in $k_1 = C_{f1}\sqrt{\mathrm{Re}}$ in the equilibrium boundary layer at the cone for $M_\infty = 25.7$ (a) and for $M_\infty = 41$ (b). Numbers 1, 2 and 3 denote the results along the cone generatrices corresponding to $\eta = \pi / 20$, $\eta = \pi / 2$, $\eta = \pi$.

The quantity $k_2 = (1 / \sigma_e)\ \mathrm{Nu} / \sqrt{\mathrm{Re}}$ which characterizes the heat flux at the wall, is written identically for the equilibrium layer (4.2) and the frozen layer [4], however the coefficient P_1^* and the controllong functions (4.3) all depend on the model of air used.

Figure 5 depicts the coefficient k_2 for $M_\infty = 25.7$ (a) and $M_\infty = 41$ (b). Computations show that in the case in which the dissociation reactions are essential $(M_\infty = 25.7)$ the influence of the choice of the gas model on k_2 is insignificant. When the speed of flight increases, the gas begins to ionize and the choice of the model begins to have an effect on the value of k_2. In the equilibrium gas k_2 becomes smaller. The dashed lines show the values of the coefficient k_2 for the frozen boundary layer at the cone surface. The diversity increases on moving away from the windward spreading line. The total convective flux towards the wall (1.10) is

$$I_q = -(H_e - H_w)\sqrt{\mu_e\rho_e}\sqrt{\frac{u_e}{a}}\sqrt{\delta^{(0)}}\ P_1^*(0.234 + 0.209d^{(0)})$$

Thus the heat flux towards the wall in a flow of gas past a body is determined by the drop of the total enthalpy, the variation in the parameter $\mu_e\rho_e$, change in the velocity of the outer flow along the body surface, the geometry of the body and the choice of the gas model.

REFERENCES

1. Suslov, O.N. Tirskii, G.A. and Shennikov, V.V., Flows of multicomponent ionized mixtures in chemical equilibrium described within the framework of the Navier—Stokes and Prandtl equations. PMTF No. 1, 1971.

2. Shevelev, Iu. D., Difference computation methods for three-dimensional laminar boundary layer. Coll.; Some Applications of the Net-Point Method in Gas Dynamics. No. 1, Flows in the Boundary Layer, Izd. MGU, 1971.

3. Shakhov, N.N. and Shevelev, Iu.D., The method of successive approx-

imations in problems of three-dimensional laminar boundary layer. PMM Vol. 38, No. 5, 1974.

4. Kazeikin, S. N. and Shevelev, Iu. D., Three-dimensional boundary layer in a partly ionized multi-component gas. PMM Vol. 40, No. 3, 1976.

5. Fay, J. A. and Riddel, F., Theoretical analysis of heat exchange at the frontal point in dissociated air. In coll.: Problems of Motion of the Nose Cone of Long-Range Rockets. M., Izd. Inostr. Lit. Moscow, 1959.

6. Suslov, O. N., Multicomponent diffusion and heat exchange in a flow of an ionized gas in chemical equilibrium past a body. PMTF, No. 3, 1972.

7. Numerical Analysis of Current Problems of Gas Dynamics. Moscow, "Nauka", 1974.

Translated by L. K.